Sigmoid Functions

We define a *sigmoid function* to be a bounded, monotonically increasing real-valued function defined on the real line. That is, a sigmoid function is a mapping

$$\boldsymbol{S}: (-\infty, +\infty) \to (\mathbf{a}, \mathbf{b}),$$

where **a** & **b** are finite, and for all $\mathbf{x}$ & $\mathbf{y} \in (-\infty, +\infty)$, if $\mathbf{x} < \mathbf{y}$ then $\boldsymbol{S}(\mathbf{x}) \leq \boldsymbol{S}(\mathbf{y})$.

Generic

One example of a sigmoid function is the *logistic* mapping

$$\boldsymbol{S}: (-\infty, +\infty) \to (0, \mathbf{L})$$

with $\mathbf{L} > 0$, defined by

$$\boldsymbol{S}(\mathrm{x}) = \frac{\mathbf{L}}{1 + e^{-x}},$$

with the first derivative

$$\boldsymbol{S}'(\mathrm{x}) = \frac{\mathbf{L} \cdot e^{-x}}{(1 + e^{-x})^2}.$$

tanh(x)

Another example of a sigmoid function is the hyperbolic tangent function

$$\mathbf{tanh}: (-\infty, +\infty) \to (-1,1)$$

defined by

$$\mathbf{tanh}(\mathrm{x}) = \frac{e^{2x} - 1}{e^{2x} + 1},$$

with the first derivative

$$\mathbf{tanh}'(\mathrm{x}) = 1 - \left(\mathbf{tanh}(\mathrm{x})\right)^2.$$